Analysis of Distributed Resource Allocation in MIMO Systems Using Game Theory

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Abstract—Research has shown that efficient and effective radio resource management is a key requirement for improved capacity and quality of service in wireless communication networks. Most resource management schemes are based on centralized frameworks where a central resource manager performs all the resource allocation functions. Centralized schemes have been proven to be inefficient especially in modern wireless networks employing the MIMO technology. Distributed schemes provide a better alternative; the resource allocation decisions are performed by all the nodes in the network. However, the interactive decision making process complicates the decision making resulting in conflicts among the various decision makers. Resolving such conflict using traditional optimization methods is not only difficult, but also compromises fairness. Game theory has been proposed before for such problems. This paper presents an analysis of a distributed channel allocation in multi user MIMO wireless system using game theory. We formulate a non-cooperative game for the channel allocation problem and analyze its convergence to the Nash Equilibrium (NE).

Index Terms— Game theory, MIMO, Resource Allocation

I. INTRODUCTION

The increase in the use of wireless networks calls for the design of efficient measures of managing the scarce radio resources such as frequency channels and power. In order to achieve high spectral efficiencies in these networks, there is a need for proper mitigation and management of the overall interference in the wireless networks as a result of sharing of the same wireless channels among several users in the system. The interference may occur in a case where transmitters and receivers in the network are randomly placed and interfere with their neighbors. Efficient resource management schemes should provide a framework for allocating the available wireless channels at each transmitter in the network so as to increase the system capacity while achieving the required quality of service (QoS) objectives.

MIMO technology, which involves the use of multiple antennas at the transmitter and receiver of a system, has been shown to provide improved performance in terms of diversity and high throughput without incurring any additional expenditure on power and bandwidth [1], [2]. Transmission of signals over multiple independently fading spatial paths mitigates multipath fading resulting in a diversity gain. Also, transmitting independent data signals over multiple antenna streams can increase the system capacity as a result of multiplexing gain. Processing of signals at both the transmitter and the receiver may result in some form of coherent combining effect hence increasing the system’s average signal to noise ratio (SNR) [1], [2], [3].

Resource allocation schemes can broadly be classified as either fixed or dynamic. In fixed schemes, the resource allocation procedure is only performed once, normally before the network deployment. Once the initial allocation is done, the radio resources cannot be re-allocated again [4]. In dynamic schemes, the allocation decisions vary with the varying network conditions.

Dynamic schemes can either be centralized or distributed. Centralized schemes involve the use of a single central station/node (resource manager) to perform all the allocation decisions [5]. The active nodes in the system measure all the network information (channel gains to other nodes) and pass it to the resource manager who determines the final resource allocations. In distributed schemes, each transmitter determines its allocation independently based on the information from other transmitters. Therefore, network nodes need to interact with each other when making the allocation decisions. The need for interactions between nodes brings about some conflicts in the resource allocation process as selfish nodes may increase their transmit power without considering the neighbor nodes hence causing interference to them. The neighboring nodes may respond
by increasing their transmit powers as a counter measure thereby degrading the overall system performance. Such an interaction can be modeled as a resource allocation problem with specific optimization goals ranging from the maximization of system throughput, ensuring the quality of service (QoS) for each user, minimizing the transmit powers and so on.

Different analytical methods have been used to solve the resource allocation problem in MIMO wireless networks [6], [7], [8]. However, these methods fail to resolve the conflicts arising from node interactions fairly while achieving the optimization objectives. The analysis of such interactions using traditional analytical optimization methods has been described as difficult hence justifying the use of game theory [8], [9].

With a proper definition of the utility functions in a game, the best strategy response can be shown to exist for every player. Game theory has been used in recent past to study several radio resource management problems. A game theoretic framework for resource allocation in multiuser MIMO was presented in [10]. The aim is to maximize the throughput and energy efficiency in the system. The authors in [11] present a cooperative game theoretic framework for resource allocation for multi user MIMO-OFDMA downlink with an aim of attaining a trade-off between spectral efficiency and user fairness. Power allocation in MIMO systems has also been extensively studied e.g. in [17], [18] and so on.

In this paper, we analyze the power allocation problem in a multi-user MIMO wireless system using non-cooperative games. In our analysis, we formulate the power allocation game and analyze the various convergence options to the optimal solution. The paper can be summarized under the following two objectives:

- Formulation of a non-cooperative game model for a distributed power allocation in multi user MIMO system.
- Mathematical analysis of the convergence of the game to optimal solution, which is the Nash Equilibrium point of the non-cooperative game.

The rest of the paper is organized as follows: Section II presents the basics of Game theory, section III presents the MIMO system model, section IV presents the game formulation, section V gives a detailed analysis of game and the Nash Equilibrium and section VI presents the conclusion.

II. GAME THEORY BASICS

Game theory is a field of applied mathematics that is used to study and analyze interactive decision making processes [9], [12], [13]. It has the capability to predict analytically, the outcome of complex and challenging interactions among rational decision makers. The decision making process is normally governed by two assumptions; Rationality of the individuals involved and their ability and the capability to reason strategically. Rationality translates to the ability to adhere strictly to some defined strategies based on measured results. The decision makers (players) are therefore required to choose their own strategies according to their own preferences while considering the preferences of the other individuals.

Games can either be represented as normal/strategic form or extensive form games. The normal form game is the most common and will be considered in this study. It is usually represented in the form [5]:

\[ G = \left\{ N, \{A_i\}_{i=1}^{N}, \{u_i\}_{i=1}^{N} \right\} \]  

(1)

Where \( N = \{1, 2, \ldots, n\} \) represents the vector of players in the game, \( A_i \) is the actions available to a player \( i \). The actions available to each player can be represented in form of a Cartesian product; \( A = A_1 \times A_2 \times \ldots \times A_n \). \( \{u_i\} = \{u_1, u_2, \ldots, u_n\} \) represents the utility functions for a player \( i \). In optimization theory, these functions would be considered as the objective functions that each player would wish to minimize or maximize.

Generally, the utility function \( u_i \) represents the actions chosen by player \( i \), \( a_i \) and the actions chosen by the other players in the game, other than player \( i \), \( a_{-i} \). The action tuple, \( a \) constitutes the action sets \( a_i \) and \( a_{-i} \). The utility function is normally given as a real valued function; \( U_i : A \rightarrow \mathbb{R} \). Common examples of the normal/strategic form games are the Prisoners’ Dilemma game and The Battle of the Sexes.

III. SYSTEM MODEL

We consider a MIMO system where both the transmitters and the receivers are equipped with multiple antennas ( \( N_t \) and \( N_r \) respectively) as shown in figure 1. A spatially independent, identical, distributed (i.i.d) flat fading channel is assumed whose channel coefficient matrix for a user \( i \), \( H_i \) is given by \( N_t \times N_r \). Let the channel information be available at the receivers (CSIR). We also assume a system with low complexity receivers with no interference cancellation capabilities. This means that the receivers treat the interference from other users as additive spatial noise. The received signal at receiver \( k \) is given by:

\[ y_k = H_n x_i + \sum_{i \neq j} H_n x_j + n_i \]  

(2)

Where \( x_i \in \mathbb{C}^{N_t \times 1} \) is the transmit signal represented as a vector, \( H_n \) is the flat fading channel coefficients matrix and \( n_i \) is the complex Additive White Gaussian Noise (AWGN) at receiver \( k \) which is circularly symmetric with mean of zero and variance \( \sigma^2 \). Every transmitter in the system has a maximum power constraint of \( P_i \). The transmit covariance matrix associated with the transmitter \( i \) is given by:

\[ Q_i = E\left[ x_i^H x_i^H \right] \]  

(3)
This implies that:
\[ E\left[\|x_i\|^2\right] = \text{Tr}(Q_i) \leq P_i \]  (4)

The maximum achievable information rate, \( R \) on a link
\( i \) can be expressed as a function of the transmit covariance
matrix \( Q_i \).
\[ R_i(Q_i, Q_j) = \sum_{i,j=1}^{N_t} \left\{ \log \det \left( I + H_i^H \psi_i^{-1}(Q_j) H_j Q_j \right) \right\} \]  (5)

Where \( \psi_i^{-1}(Q_j) \) is the noise plus interference covariance
matrix as a result of the adjacent transmitters and is given by:
\[ \psi_i^{-1}(Q_j) = \sigma^2 I + H_i Q_j H_i^H \]  (6)

![MIMO system representation](image)

**Figure 1: MIMO system representation**

### IV. NON-COOPERATIVE GAME FORMULATION

Non-cooperative games are characterized by the unwillingness of the players to cooperate in achieving the objectives of the game [12], [13]. The self-interested players in such games make selfish decisions so as to maximize their respective payoffs at the expense of the other players. The selfish behavior by such players may lead to suboptimal equilibrium in a system as undesirable steady states can be achieved [9]. Certain measures are defined to check the behavior of the selfish players in non-cooperative games. Pricing is one such measure that discourages the selfish actions among players. Incentive mechanisms can also be used to encourage players to cooperate when choosing their actions. The main types of incentive mechanisms in literature can be grouped into two main categories: credit-exchange based and reputation based mechanism. The operation of these mechanisms is however beyond the scope of this paper.

The Nash Equilibrium (NE) is usually defined for non-cooperative games to represent the most viable operating point where no single player can gain from unilaterally changing its strategy when the other players remain with the current strategies. The NE point is usually unique but may sometimes be socially undesirable in a system in that, it may not necessarily represent the optimal point. The concept of Pareto optimality is used to compute a measure of efficiency of the NE.

We now model the power allocation scheme as a normal form game. For simplicity, let us assume a two user system. The word user and player are used interchangeably in this section to mean the same thing. The game takes the following form:

\[ G = \left\{ K, \{S_k\}_{k \in K}, \{U_k\}_{k \in K} \right\} \]  (7)

\( K \) represents the players in the game which is basically the total number of users in the formulated game, \( \{S_k\} \) is the strategy set for the player \( k \). \( \{U_k\}_{k \in K} \) is the utility function of every player and is usually defined as a real-valued function.

The achievable rates for the two users on link \( i \) are:
\[ R_i(Q_i, Q_j) = \log \left\{ \det \left( I + H_i^H \psi_i^{-1}(Q_j) H_j Q_j \right) \right\} \]  (8)

And
\[ R_2(Q_i, Q_j) = \log \left\{ \det \left( I + H_2^H \psi_2^{-1}(Q_1) H_1 Q_1 \right) \right\} \]  (9)

These rates can be achieved within a specific region which is expressed as a function of the transmit covariance matrices. The region for user 1 is:
\[ S_1 = \left\{ Q \succeq 0 : \text{Tr}(Q) \leq P_i \right\} \]  (10)

\( S_1 \) is the strategy space available to user 1 and is given by:
\[ S_1 = \left\{ Q \succeq 0 : \text{Tr}(Q) \leq P_i \right\} \]  (11)

The achievable rate region and strategy space for user 2 takes the same form as for user 1.

The normal game representation for this system can then be given as follows:
\[ G_{2\text{players}} = \left\{ \{K_i\}_{i=1,2}, \{S_i\}_{i=1,2}, \{R_i\}_{i=1,2} \right\} \]  (12)

### V. GAME ANALYSIS AND NASH EQUILIBRIUM (NE)

The NE is used to describe the most viable operating point in a non-cooperative game [10], [12], [14]. This point may not be necessarily unique in the power allocation problem in MIMO as explained in [15]. We use Iterative Water-Filling Algorithm to analyze the power allocation problem in multi user MIMO guaranteeing the uniqueness of the NE. Every user computes the optimal transmit covariance matrix \( Q_{\text{opt}} \) with respect to its own channel while considering the channel contributions from the other users as noise.
\( Q_{i,\text{opt}}(Q_2) = U_i \left( \mu I - D_i^{-1} \right)^T U_i^H \) \hspace{1cm} (13)

Where \( \mu \) is always chosen to satisfy the condition \( \text{Tr} \left( \mu I - D_i^{-1} \right)^T = P_i \) and \( U_i \) and \( D_i \) are computed from the EigenValue decomposition of the channel which is represented by;

\[
H_i^T \psi_i^{-1}(Q_2) H_i = H_i D_i U_i^H
\]

The second user computes his \( Q_{\text{opt}} \) by following the same procedure assuming the other user’s channel as noise.

A. Nash Equilibrium: Existence and uniqueness

The convergence of the iterative water filling algorithm to the NE is guaranteed under the following condition [16]:

\[
[S]_{ji} = \begin{cases} 
\rho \left( H_i^T H_i^{-1} H_j^T H_j \right) & j \neq i \\
0 & \text{otherwise}
\end{cases}
\]

For any set of channel matrices and transmit power of users in a MIMO system, there exist at least one NE if \( \rho(S) > 1 \).

\( \rho(S) \) is the spectrum of matrix \( S \).

VI. CONCLUSION

This paper presented an analysis of resource allocation in multi user MIMO wireless system using strategic non-cooperative game. The power allocation problem was analyzed by use of an iterative water filling algorithm. The condition for existence and uniqueness of the Nash equilibrium (NE) point in the game was also presented. The existence of the NE in the game does not necessarily translate to the optimal power allocation point in the system because it may be socially undesirable. Its efficiency can therefore be measured through Pareto optimality test. This efficiency can also be depicted through what is commonly known as the Price of Anarchy (POA). However, this phenomenon was beyond the scope of this paper. The paper presented a pure mathematical analysis of the resource allocation problem. Simulations of the analysis for validation purposes will be conducted in future works.

VII. REFERENCES


Danson G. Njue received his undergraduate degree in 2005 from the Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya and is presently studying towards his Master of Science degree at Tshwane University of Technology. His research interests include wireless networks, Game Theory, Optimization and Radio Resource Management in Wireless networks.