A Low Complexity Greedy Scheduler for Multiuser MIMO Downlink

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Abstract—It is widely believed that the implementation of multi-user multiple input multiple output (MU-MIMO) technology at the radio access portion of current and future wireless networks would positively impact on the performance of such networks especially in terms of system capacity. This however demands the design of efficient multi-user scheduling algorithms at the data link layer. In the case of packet data which is delay tolerant, there is more flexibility in the design of such multi-user scheduling algorithms. One such algorithm that is known to be throughput optimal is the greedy scheduler. For the downlink of a single input single output (SISO) system, the greedy scheduler serves the user whose channel maximizes the channel capacity at each transmission opportunity. By employing this algorithm to the downlink of a MU-MIMO system applying spatial multiplexing, the scheduler can allow a subset of active users whose channels are most favourable to transmit simultaneously. However, unlike in the SISO systems, the implementation of the classical greedy scheduler in MU-MIMO systems would result in very high computational complexity. This paper proposes a low complexity greedy scheduler for the MU-MIMO downlink and through simulations, demonstrates that the low complexity greedy scheduler performs close to the classical greedy scheduler but with minimum complexity.

KEYWORDS: MU-MIMO, Scheduling, Opportunistic.

I. INTRODUCTION

One of the major applications of digital communications is wireless communications. The mobile radio channel dominates the design of wireless communications and hence most of research on wireless communications has always been centered on the mobile radio channel. With such intensive research on the mobile radio channel, we have had many developments with a current example being that of Multiple Input Multiple Output (MIMO) systems. MIMO systems can be defined simply as a communications system that is equipped with multiple antennas at both the transmit and receive sides. With such an arrangement, the multiple antennas at both ends of a wireless link can offer significant performance gains by use of spatial diversity and/or spatial multiplexing. Spatial diversity allows for redundant transmission or reception of the same signal on multiple antennas thus improving the link quality as is the case with other diversity techniques such as time or frequency diversity. On the other hand, spatial multiplexing allows for multiple transmission of different data streams to a single user (Single-user MIMO) or to multiple users (MU-MIMO) increasing the data rates or the number of users respectively [3, 4].

Unlike SISO systems which suffer greatly from random fading, with proper antenna design, MIMO systems can exploit the random fading typical of wireless channels to improve the system performance [5]. With typical frequency bandwidth constraints present in wireless links, this was an important breakthrough since we can improve the performance of wireless networks with no extra cost on bandwidth. Observing that the implementation of MIMO technology at the radio access portion of current and future wireless networks would positively impact on the performance of such networks especially in terms of system capacity [2, 3], the design of an efficient scheduling algorithm is required at the data link layer of MU-MIMO systems to efficiently capitalize on the benefits presented by this technology [12]. In this work, we intend to design a low complexity greedy scheduler for the downlink of a MU-MIMO system where data packets for different users are spatially multiplexed.

From [8], an efficient scheduler that would increase the link capacity of multi-user wireless systems would be the one that exploits multi-user diversity (MUD) inherent in a typical wireless channel. MUD is a phenomenon whereby, in a multi-user system, each user experiences independent fading characteristics. In such a scenario, an efficient scheduling algorithm that is throughput optimal would be the one that serves the user achieving maximum rate at each transmission opportunity [8]. This ensures that system resources are only allocated to the user that best exploits the wireless link (offers maximum link capacity). This is known as opportunistic or greedy scheduling, and is known to offer significant performance in terms of link capacity where users have infinite backlogs and with users’ channels that are symmetrical [6].

For purposes of this work we consider the multi-user scheduling problem of a MIMO downlink with users that are infinitely backlogged. We further assume a system whose channel is both perfectly known at the transmitter and receiver. The greedy scheduler in [8] can be modified for a MU-MIMO system such that in each scheduling interval, the subset of active users with the most favourable channels
are scheduled. Such a solution would be throughput optimal but with high computational complexity since the scheduler would have to search for all the possible combinations of users on the transmit antennas out of the total number of active users. The focus of this work is to come up with a scheduling algorithm that performs close to the optimal classical greedy scheduler with reduced computational complexity. The rest of the paper is organised as follows. In section II, we present the MU-MIMO downlink model with $K$ active users. Section III presents the MU-MIMO greedy scheduling algorithm that is throughput-optimal and we derive the low complexity greedy scheduler. Section IV provides the complexity analysis of the two algorithms while we present the simulation results and conclude in sections V. and VI respectively.

II. MU-MIMO SYSTEM MODEL

The downlink of a typical MU-MIMO system composed of a base transceiver station (BTS) with $N_{TX}$ antennas and $K$ mobile stations (MSs), each with $N_{RX}$ antennas is considered. We assume that spatial multiplexing is applied at the BTS, allowing for simultaneous transmission to at most $N_{TX}$ multiple users. We further assume that the transmit antennas at the BTS transmit with equal power. A quasi-static channel is assumed from the BTS to each MS implying that the channel (and by extension the users’ rates) from the BTS to each MS varies from timeslot to timeslot but is static for the timeslot duration. We further assume that at any one time, a single BTS antenna communicates to only one user.

At each time $t$, the receive signal of user $k$ is given as a vector of form

$$\mathbf{y}_k(t) = \mathbf{H}_k \mathbf{s}(t) + \mathbf{n}_k(t)$$  \hspace{1cm} (1)

where $\mathbf{H}_k$ is a matrix representing the $N_{RX} \times N_{TX}$ channel matrix from the BTS to user $k$, $\mathbf{s}(t)$ is a vector representing the $N_{TX} \times 1$ signals transmitted by the BTS, and $\mathbf{n}_k(t)$ is a vector representing additive white Gaussian noise (AWGN) at the receiver of user $k (k=1,\cdots,K)$. The noise covariance matrix is given by $\sigma^2_{SNR} \mathbf{I}_{N_{RX}}$ where $\mathbf{I}_{N_{RX}}$ is an identity matrix of order $N_{RX}$. We assume that the channel from the BTS to the respective users suffers from shadow fading with a log-normal distribution, path loss fading, and multi-path fading. At each timeslot, assuming there is no spatial correlation at the receive antennas, $\mathbf{H}_k$, the channel matrix between the BTS and a user $k (k=1,\cdots,K)$ is expressed as $[10, 12, 13].$

$$\mathbf{H}_k = \sqrt{SNR_0 (d_B/D)^{-\beta} \times 10^{\delta_k/10}} \times \mathbf{G}_k,$$  \hspace{1cm} (2)

where $\mathbf{G}_k$ is an $N_{RX} \times N_{TX}$ matrix whose entries have a Rayleigh distribution with a mean of zero and unit variance, representing multi-path fading. $S_2$ is a real Gaussian random variable with mean of zero and a variance of $\sigma^2_{SNR}$. $\beta$ denotes the path loss exponent, $d_B$ denotes the distance between user $k$ and the BTS, $D$ represents the cell radius with $(d_B \leq D)$ and $SNR_0$ represents the median signal to noise ratio (SNR). Due to simultaneous transmission of multiple data streams (spatial multiplexing), every receiver suffers from multi-stream interference (MSI) and we need to effectively perform interference mitigation on each of the receivers. This is done with proper design of the receiver structures. For optimal performance, the maximum likelihood (ML) receiver would be the number one choice but it is more complex especially as the number of antennas increases $[7, 12].$ Without loss of generality, we can opt for the lesser complex linear receivers namely; the minimum mean square error (MMSE) or the zero-forcing (ZF) receivers. Any of this would offer close to optimal performance with relatively low complexity. Since the MMSE is known to outperform the ZF in terms of balancing the interference mitigation with noise enhancement $[7, 13]$ and since the choice of the receiver structure does not affect the performance of our proposed scheduling algorithm, we assume the implementation of the MMSE receiver at each MS.

Employing the MMSE at the receivers, the vector of signals for user $k (k=1,\cdots,K)$ at time $t$ is given as

$$\mathbf{z}_k(t) = \mathbf{W}_k \mathbf{y}_k(t) = \mathbf{W}_k \mathbf{H}_k \mathbf{s}(t) + \mathbf{W}_k \mathbf{n}_k(t),$$ \hspace{1cm} (3)

with $\mathbf{W}_k$ representing the MMSE weight matrix which is constant for each scheduling interval and is calculated as

$$\mathbf{W}_k = ([\mathbf{H}_k^H \mathbf{H}_k + (\sigma^2_{SNR} / \Phi) \mathbf{I}_{N_{RX}}])^{-1} \mathbf{H}_k^H,$$ \hspace{1cm} (4)

with $(\cdot)^H$ denoting the conjugate transpose and $\mathbf{I}_{N_{RX}}$ the identity matrix of order $N_{TX}$. From $[11, 12]$, the SINR for a user $k$ on channel $\tau$ ($user k (k=1,\cdots,K)$ assigned transmit antenna $\tau$ ($\tau = 1,\cdots,N_{TX}$)) is calculated as

$$\gamma_{k,\tau} = \frac{||\mathbf{W}_k \mathbf{H}_k||^2}{\sigma^2_{SNR} \sum_{m=1}^{N_{TX}} ||\mathbf{W}_m||^2 + \sum_{m=1, m \neq \tau}^{N_{RX}} ||\mathbf{W}_m \mathbf{H}_k||^2},$$ \hspace{1cm} (5)

with $\Phi$ denoting the total transmit signal power from all the transmit antennas.

Assuming the employment of rate adaptation at the BTS, the actual rates calculated using the respective SINR’s for user $k$ on channel $\tau$ can be mapped on to the discrete rates. If we employ uncoded M-ary $(M = 2^j, j = 1, 2,\cdots)$ modulation schemes for the transmissions and since for uncoded M-QAM the throughput curves form an envelope that is parallel to the curve of the Shannon capacity given by $log_2(1 + \gamma_k)\Phi$ with a fixed offset of about 8 dB $[1]$, a user $k$ rate on channel $\tau$ can be given by $[12, 13].$

$$r_{k,\tau} = B \times \min(8, [log_2 (1 + \gamma_{k,\tau} / \Phi)]),$$ \hspace{1cm} (6)

with $B$ representing the bandwidth in Hz and $10\log_{10}\Phi = 8$ dB. The 8 in equation (6) is under the assumption that for each channel $\tau$ employing a practical uncoded modulation scheme, $j = 8$ is the maximum attainable spectral efficiency. This is equivalent to an uncoded modulation scheme with $M = 2^8$ corresponding to uncoded 256-QAM. Note that for simplicity and without loss of generality, only the rates exceeding the maximum achievable rate under an uncoded 256-QAM modulation scheme are mapped onto the discrete rate with 8 as the spectral efficiency.
III. MU-MIMO SCHEDULING ALGORITHMS

A. Greedy Scheduler

The Greedy Scheduler (GS), also known as the Maximum Rate Scheduler (MRS) exploits variations in the time varying wireless channel. The selection metric is the channel capacity which allows for the selection of a user whose channel provides the maximum rate to be served at any transmission opportunity [8]. In other words, the selected user \( k^*_i \) at the \( i \)th transmission opportunity is determined as:

\[
k^*_i = \arg \max_k R^k_i,
\]

with \( R^k_i \) denoting the channel capacity of user \( k \) at transmission opportunity \( i \). For the multi-user MIMO case, the scheme can be given as follows: Assuming that we have \( K \) active users at any given transmission opportunity, and that there is full channel information at the transmitter (CSIT), then the BTS can calculate users’ rates \( r_{k,\tau} \) using equation (6). The selected users at the \( i \)th transmission opportunity will be determined as:

\[
(k^*, \tau^*) = \arg \max_{k \in K, \tau = 1, ..., N_{TX}} r_{k,\tau}
\]

with \( r_{k,\tau} \) representing the channel capacity of user \( k \) being served by transmit antenna \( \tau \) at the current transmission opportunity. This means the BTS has to compute the achievable rates for all the possible user combinations on the \( N_{TX} \) antennas which is given by \( c^k_{N_{TX}} \). This brute search is optimal but the complexity is too high.

B. Low Complexity Greedy Scheduler

The proposed Low Complexity Greedy Scheduler (LCGS) is a simplification of the GS. As in the GS, full channel information is assumed at the transmitter and at the start of every transmission opportunity, it computes the rates of all the \( K \) users\( r_{k,\tau} \) as in equation (6). However, instead of performing a brute search for all possible user combinations, the BTS only picks the user antenna pair that offers the maximum rate. This is under the assumption that each user aims to achieve maximum rates and would rather be served by the transmit antenna that maximizes its rate. This way, the scheduling problem for a MU-MIMO system having \( N_{TX} \) antennas is effectively reduced to a scheduling problem of \( N_{TX} \) SISO system and the BTS thus applies (7) for all the \( N_{TX} \) antennas. This effectively reduces the computation complexity compared to that of the GS.

IV. COMPLEXITY ANALYSIS

The main focus of this work is to design a MU-MIMO scheduling algorithm that is capable of performing close to the GS algorithm in terms of link capacity while keeping its complexity at a minimum. In section III., we proposed the LCGS as a possible candidate to achieving the above-mentioned goal. In this section, we mathematically analyse the complexities of the two algorithms using the idea of flop counts. In [9] the author defines a flop, \( \psi \) as a real floating point operation such as a real multiplication, addition, division or subtraction. The author also defines a complex multiplication/division and a complex addition/subtraction as having six flops and two flops respectively. For purposes of this paper, we can use such a quantification procedure that gives an idea of the computational complexity involved in the two algorithms. For the MU-MIMO system described in section II. above, it means that for both algorithms, we have to calculate \( N_{TX} \) achievable rates for each user as in equation (6) since each user achieves a different rate for each channel \( \tau \). For simplicity, we can ignore the user-rate calculations since we have same complexities in the two algorithms. Focussing on user scheduling for \( K \) users with \( K \geq N_{TX} \), the complexities for the two algorithms can be computed as follows:

A. Greedy Scheduler

The algorithm for GS goes as follows (note we already have the rates):

1. Compute all the different combinations of \( K \) users on the \( N_{TX} \) antennas which is calculated as

\[
\frac{K!}{N_{TX}!(K-N_{TX})!}
\]

Complexity in terms of flops for the first step is computed as

\[
\psi_{GS1} = (K - 1) + (N_{TX} - 1) + (K - N_{TX} - 1) + 1
\]

2. Get the sum of all the rates in each combination of users. The complexity in terms of flops for step 2 is given by,

\[
\psi_{GS2} = \frac{K!}{N_{TX}!(K-N_{TX})!} \times (N_{TX} - 1)
\]

3. Sort the sum-rates computed in step 2 in descending order and pick the first combination as the group with the highest sum-rate. Complexity in terms of flops is given by,

\[
\psi_{GS3} = \frac{K!}{N_{TX}!(K-N_{TX})!} - 1
\]

The total complexity for the GS algorithm can be computed for different number of users \( K \) with \( N_{TX} \) antennas as:

\[
\psi_{GS} = \psi_{GS1} + \psi_{GS2} + \psi_{GS3}
\]

B. Low Complexity Greedy Scheduler

The algorithm for LCGS goes as follows (note we already have the rates):

1. Form a \( K \) rows by \( N_{TX} \) columns matrix with rows representing the users and columns representing the antenna index, with the entries of the matrix given by the rates as in equation (6). There will be no computation complexity associated with step 1.

2. For each row, leave out the maximum column entry and make the other column entries equal to 0. This gives the antenna index that maximizes each user’s rate. The complexity in terms of flops for step 2 will be:

\[
\psi_{LCGS2} = K (N_{TX} - 1)
\]
3. For each column, select the maximum entry. This gives the user index for each antenna that maximizes the system capacity. Complexity for step 3 in terms of flops is:

\[ \psi_{LCGS}^{(3)} = N_{TX} (K - 1) \]  

(15)

The total complexity for the LCGS algorithm can be computed for different number of users \( K \) with \( N_{TX} \) antennas as:

\[ \psi_{LCGS} = \psi_{LCGS}^{(2)} + \psi_{LCGS}^{(3)} \]  

(16)

V. SIMULATION RESULTS AND DISCUSSIONS

A. System Parameters

For our simulation, we used a \( 4 \times 4 \) MU-MIMO system. We assume the median signal to noise ratio, \( SNR_o \) of 8 dB, the standard deviation of log-normal shadowing, \( \sigma_S^2 \) is assumed to be 8 dB, the path loss exponent is assumed to be 4 dB, the cell radius, \( D \) is fixed at 1 Km, and we assume \( P_T/\sigma_N^2 \) to be 10 dB. Finally, we assume that the distance \( d_k \) between user \( k \) and the BTS at the start of each timeslot is a random value between 0 and 1 Km which implies that the users move randomly within the cell but the scheduling intervals are too small for them to be assumed fixed for each transmission opportunity. We assume the bandwidth \( B \) to be 5 MHz in (6). Using (6), the maximum achievable rates for four scheduled users is 160 Mbps.

B. System Performance

The two scheduling schemes, GS and LCGS were compared and the results were as follows: Figure 1 compares the rates achieved with 20 active users for a duration of 200 timeslots by the two schemes. It can be observed that the rates closely follow each other but with a few cases of very low rates in the case of the LCGS. This is expected because you might end up having many number of users competing for the same antenna(s) with other antennas having less competing users. In figure 2 we have the same set-up as above but with number of active users increased to 40. In this case, the two schemes have the curves a bit more closer with increased minimum rates for the LCGS. This is because with more users, the probability of having all the four antennas being preferred by users increases and also, the probability of having some users with favourable channels increases. This is emphasized by the fact that both the curves on average exhibit higher capacities as opposed to the case with 20 active users.

To effectively compare the two schemes, figure 3 shows the average capacities of 10 to 40 active users averaged over a duration of 200 timeslots while increasing the number of active users by 2 in each experiment. From the two curves it can be observed that on average, the LCGS performs very close to the optimal GS, especially with increasing number of active users. Looking at figures 3 and 4, it can be noted that with increase in number of active users, the complexity of the GS scheduler increases while its performance compared to the LCGS reduces. This implies that with large numbers of active users, it’s better to use the LCGS. For instance, figure 4 shows complexity versus number of active users from 10 to 40. Interestingly, with 40 active users, the LCGS has a performance of about 97.02% with only about 0.0755% complexity (in flops) as compared to the GS algorithm.

VI. CONCLUSION

This work aimed to design a low complexity scheduling algorithm in the multi-user MIMO downlink. We first established that the classical greedy scheduler is capacity optimal since for each transmission opportunity, it schedules a subset of the active users to maximize the MIMO link capacity. It is however noted that to perform a complete search in all the possible user-antenna combinations demands a high computational complexity. From our simulations, it is demonstrated that a near optimal scheme, in this case the low complexity greedy scheduler performs close to the greedy scheduler and especially with large numbers of users. From this work, we can comfortably suggest that there exists some MU-MIMO scheduling algorithms with low complexities...
that would perform close to optimal schemes. It is thus in the
interest of future work to investigate on complexity versus
performance tradeoff for existing/proposed MU-MIMO
scheduling algorithms.

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