The Effect of Signal-to-Noise margins on the performance of ADSL

J.H. van Wyk and L.P. Linde

Abstract—There is renewed interest in existing copper networks since the introduction of Asymmetric Digital Subscriber Line (ADSL) technology. ADSL will soon be available in South Africa, opening the broadband market to prospective users. As many users are placed within the same cable, they interfere with each other. Service providers want to guarantee the supplied data rate to the customer, even as the network change over time. For this purpose, a Signal-to-Noise ratio (SNR) margin is added to the customer profile.

This paper estimates the number of interferers (possible services) which can be allowed for different line lengths, interferer types and SNR margins for a maximum data rate over the line.

Keywords—Asymmetric Digital Subscriber Line (ADSL), Signal-to-Noise ratio (SNR) margin, Performance margin, Interference margin, Margin estimation

I. INTRODUCTION

The loop plant (lines radiating from the exchange to customers) is the only part of the telephone network that stayed structurally almost the same for the better part of a century. It primary use was for basic voice communication, but the introduction of broadband technology and applications led to a transformation of the network. Nowadays the network incorporates various broadband technologies, of which Asymmetric Digital Subscriber Line (ADSL) will soon be part. Although considered as a point-to-point service, interference between services occur within the cable. Spectral management must be performed by service providers to guarantee a certain data rate to users, even if the network should degrade with time. Every user’s line has a specific topology, which can be represented by a signal-to-noise (SNR) profile. If a certain margin is added to this profile, the provider can guarantee the data rate to the customer. This margin, referred to as SNR margin, performance margin or interference margin, is the topic of this paper. Our simulation aims to estimate the number of possible interferers which can be allowed for a specific line length and SNR margin. The basic modelling principles used in the simulation is discussed in Section II. Section III shows the results obtained. A discussion of the results is presented in Section IV.

II. SIMULATION MODELLING

A. Channel Modelling

The resistance $R$, capacitance $C$, inductance $L$ and conductance $G$ of a copper line, at a specified frequency $f$ are determined by:

$$R(f) = \sqrt{r_{oc}^4 + a_c \cdot f^2}$$

where $r_{oc}$ is the copper DC resistance and $a_c$ is a constant characterizing the increase of resistance with frequency in the "skin effect",

$$L(f) = \frac{l_0 + l_\infty \left( \frac{f}{f_m} \right)^b}{1 + \left( \frac{f}{f_m} \right)^b}$$

where $l_0$ and $l_\infty$ are the low-frequency and high-frequency inductance respectively, and $b$ is a parameter chosen to characterize the transition between low and high frequencies in the measured inductance values,

$$C(f) = c_\infty + c_0 \cdot f^{-c_e}$$

where $c_\infty$ is the “contact” capacitance and $c_0$ and $c_e$ are constants chosen to fit the measurements, and

$$G(f) = g_0 \cdot f^{+g_e}$$

where $g_0$ and $g_e$ are constants chosen to fit the measurements. Values for the different constants for different wire types are summarized in Table I for ADSL.

The characteristic impedance $Z_o$ and the propagation constant $\gamma$ of the twisted-pair, at a specific frequency $f$ is expressed as [1,2]:

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

where $\omega = 2\pi f$.

Two-port networks, and specifically ABCD matrixes, can be used to represent the line. The ABCD parameters are related to the characteristic impedance $Z_o$ and propagation constant $\gamma$ as follows:

$$A = D = \cosh(\gamma.d)$$

$$B = Z_o \cdot \sinh(\gamma.d)$$

$$C = \frac{1}{Z_o} \cdot \sinh(\gamma.d)$$

where $d$ is the length [km] of the line segment under consideration [3].

The insertion loss function of the twisted-pair loop with source impedance $Z_s$ and terminal impedance $Z_t$ is [1,3]:

$$H_{ins}(f) = \frac{Z_s + Z_t}{A.Z_t + B + C.Z_s.Z_t + D.Z_s}$$

The attenuation through the cable [dB] is expressed as [3]:

$$L_{dB}(f) = 10 \cdot \log_{10} \left| H_{ins}(f) \right|^2$$

B. PSDs of interferers

The power spectral density (PSD) of a downstream ADSL interferer is expressed by Eq. (12) [Annex B of [4]]. $K_{ADSL-down}$ is the total transmitted power in milliwatt for a downstream ADSL transmitter before shaping filters, with $f_o$
the sampling frequency. \( LPF \) is a low-pass filter with \( f_{3dB} = 1.104 \) MHz and 36 dB/oct roll-off, expressed as:

\[
|LPF(f)|^2 = \frac{f_0^\alpha}{f_h^{\alpha} + f_0^{\alpha}}
\]

(13)

\( HPF \) is a band-pass filter with \( f_{3dB} = 4 \) kHz and 25.875 kHz with 57.5 dB attenuation in the voice band, expressed as:

\[
|HPF(f)|^2 = \frac{f_0^\alpha + f_h^\alpha}{f_0^\alpha + f_h^\alpha}
\]

\( f_l = 4kHz, \quad f_h = 25.875kHZ, \quad \alpha = 7.09 \)

(14)

The PSD of an upstream ADSL interferer is given by Eq. (15) and Table I: \( K_{ADSL-down} \) is the total transmitted power in milliwatt for a upstream ADSL transmitter before shaping filters, with \( f_o \) the sampling frequency. \( LPF \) is a low-pass filter with \( f_{3dB} = 138 \) kHz with 24 dB attenuation at 181.125 kHz, expressed as:

\[
|LPF(f)|^2 = \frac{f_0^\alpha}{f_h^{\alpha} + f_0^{\alpha}}
\]

\( f_h = 138kHZ, \quad \alpha = 20.32 \)

(16)

\( HPF \) is a band-pass filter with \( f_{3dB} = 4 \) kHz and 25.875 kHz with 57.5 dB attenuation in the voice band, expressed as:

\[
|HPF(f)|^2 = \frac{f_0^\alpha + f_h^\alpha}{f_0^\alpha + f_h^\alpha}
\]

\( f_l = 4kHz, \quad f_h = 25.875kHZ, \quad \alpha = 7.34 \)

(17)

\( HPF \) are used to separate ADSL from the POTS.

The PSD of a typical ISDN interferer is expressed by Eq. (18) [Annex B of [4]]: \( PSD_{ISDN} \) is the single sided PSD of an 80 kbaud 2B1Q signal with random equiprobable levels, with full-baud square-topped pulses with 2\(^{nd}\) order Butterworth filtering (\( f_{3dB} = 80 \) kHz).

The PSD of a typical HDSL interferer is expressed by Eq. (19) [Annex B of [4]]: \( PSD_{HDSL} \) is a single-sided PSD of a 392 kbaud 2B1Q signal with random equiprobable levels, with full-baud square topped pulses with 4\(^{th}\) order Butterworth filtering (\( f_{3dB} = 196 \) kHz).

The PSD of a typical E1 interferer is expressed by Eq. (20) [Annex B of [4]]: \( PSD_{E1} \) is the single-sided PSD of a E1 line disturber with 50% duty-cycle random Alternate Mark Inversion (AMI) at 2.048 Mbps. The transmitted pulse passes through a 3\(^{rd}\) order low-pass Butterworth filter (\( f_{3dB-Shaping} = 4 \) MHz). This shaping filter’s magnitude squared transfer function is:

\[
|H_{Shaping}(f)|^2 = \frac{1}{1 + (f/f_{3dB-Shaping})^\alpha}
\]

(21)

In addition, the coupling transformer is modelled as a high-pass filter (\( f_{3dB-Trans} = 40 \) kHz). This coupling transformer’s magnitude squared transfer function is:

\[
|H_{Trans}(f)|^2 = \frac{f^2}{f_{3dB-Trans}^2}
\]

(22)

The PSD of Near-end crosstalk (NEXT) noise, for the line under consideration, can be expressed as:

\[
PSD_{NEXT} = PSD_{Disturber} \cdot x_{n \cdot NEXT} \cdot f_{1.5}^n
\]

(23)

where \( n \) is the number of disturbers, and \( f \) is the frequency [Hz].

Far-end crosstalk (FEXT) is dependent on the characteristics of the line. The original signal at the transmitter will be attenuated due to the inherent propagation loss of the line. In a real network, FEXT is not just a function of the crosstalk in the cable, but also of the cable topology [5], i.e.

\[
PSD_{FEXT} = PSD_{Disturber} \cdot x_{n \cdot FEXT} \cdot d \cdot |H_{ins}(f)|^2 \cdot f^2
\]

(24)

where \( n \) is the number of disturbers, \( d \) is the length of the disturbing line [km], \( H_{ins}(f) \) is the insertion loss for the line under consideration, \( | \cdot |^2 \) is the modulus-squared function, and \( f \) is the frequency [Hz]. When the interfering pair is located in an adjacent binder, there is a 10 dB decrease in \( x_n \), i.e. \( x_{n \cdot NEXT} = 8.814 \cdot 10^{-14} \cdot (\frac{n}{29})^{0.6}, \quad n < 50, \quad 0 \leq f < \infty \)

(25)
\[ PSD_{ADSL-up} = K_{ADSL-up} \cdot \frac{2}{f_o} \cdot \left[ \frac{\sin \left( \frac{\pi f}{f_o} \right)}{\left( \frac{\pi f}{f_o} \right)} \right]^2 \cdot |LPF(f)|^2 \cdot |HPF(f)|^2 \]

\[ K_{ADSL-up} = 43.7mW, \quad f_o = 276kHz, \quad 0 \leq f < \infty \]  

\[ PSD_{ISDN} = K_{ISDN} \cdot \frac{2}{f_o} \cdot \left[ \frac{\sin \left( \frac{\pi f}{f_o} \right)}{\left( \frac{\pi f}{f_o} \right)} \right]^2 \cdot \frac{1}{1+\left( \frac{f}{f_{3dB}} \right)^8} \]

\[ K_{ISDN} = \frac{5}{9} V_p^2, \quad V_p = 2.50V, \quad R = 135\Omega, \quad f_o = f_{3dB} = 80kHz, \quad 0 \leq f < \infty \]  

\[ PSD_{HDSL} = K_{HDSL} \cdot \frac{2}{f_o} \cdot \left[ \frac{\sin \left( \frac{\pi f}{f_o} \right)}{\left( \frac{\pi f}{f_o} \right)} \right]^2 \cdot \frac{1}{1+\left( \frac{f}{f_{3dB}} \right)^8} \]

\[ K_{HDSL} = \frac{5}{9} V_p^2, \quad V_p = 2.70V, \quad R = 135\Omega, \quad f_o = 392kHz, \quad f_{3dB} = 196kHz, \quad 0 \leq f < \infty \]  

\[ PSD_{E1} = K_{E1} \cdot \frac{2}{f_o} \cdot \left[ \frac{\sin \left( \frac{\pi f}{f_o} \right)}{\left( \frac{\pi f}{f_o} \right)} \right]^2 \cdot \sin^2 \left( \frac{\pi f}{2f_o} \right) \cdot |H_{Shaping}(f)|^2 \cdot |H_{Trans}(f)|^2 \]

\[ K_{E1} = \frac{V_p^2}{R}, \quad V_p = 3.60V, \quad R = 100\Omega, \quad f_o = 2.048MHz, \quad 0 \leq f < \infty \]  

line under consideration.

In Eqs. (23) & (24) the condition is that \( n < 50 \). In order to allow more interferers (\( n \geq 50 \)), define two variables \( N_{50} \) and \( n_{<50} \), such that

\[ N_{50} = \left\lfloor \frac{n-50}{50} \right\rfloor \]

\[ n_{<50} = \text{mod}(n-50, 50) \]  

where \( \lfloor \cdot \rfloor \) denotes the lowest integer value of the argument. \( x_n \) can then be expressed by Eq. (26). The first term in Eq. (26) is due to the first 50 pairs being located within the same binder group. The second term is due to groups of adjacent binders, each consisting of 50 pairs. The last term is for a fraction binder, containing less than 50 pairs.

For ADSL interferers, the SNR is expressed by Eq. (27). For all other interferers, the SNR is expressed by Eq. (28). \( P_{ADSL-down} \) is the downstream transmitter power for ADSL, assumed to be -40dBm/Hz (110.4 mW). This SNR profile is then used in a bit loading algorithm [6] to calculate the data rate of the ADSL service, consisting of a 0.4mm twisted-pair line with no bridged taps.

### III. RESULTS

The maximum data rate for a 0.4mm loop, as a function of line length and SNR margin is shown in Fig. 1. For each of the interferer types (ADSL, ISDN, HDSL and E1) the number of interferers are increased till the determined data rate is equal to the maximum shown in Fig. 1. The results obtained are shown in Figs. 2-5. The specific region between a line length of 2-5 km is zoomed for Figs. 2-4, as shown in Figs. 6-8. The same region for Fig. 5 is zero for all values.

### IV. DISCUSSION

Consider Fig. 1. For a SNR margin of 0-10dB, the drop in maximum data rate can be as high as 1500 Kbps. When considering Figs. 2-5 there is a sharp decrease in the number of possible interferers as the line length is increased. This is mainly due to the increase in the resistance of the line and the presence of large FEXT interference. The region between 2.3-4.5 km is specific to the type of interferer present. As the line length is increased, there is a rise in the number of interferers, up to some maximum value. This can be explained by...
the fact that FEXT interference is reduced as the line length is increased. From this maximum value, the number of interferers decreases again to zero, mainly due to increasing line resistance. As the SNR margin is increased, there is an increase in the number of allowed interferers (services), as expected. From Fig. 8 a small number of HDSL services can be allowed. No E1 services is acceptable.

In general, all lines should be below 2 km. If not possible, the SNR margin should be chosen above 6 dB in order to allow more interferers (services).

REFERENCES


Fig. 7. Number of possible ISDN interferers as a function of line length and SNR margins (Expanded view)

Fig. 8. Number of possible HDSL interferers as a function of line length and SNR margins (Expanded view)


Jacques H. van Wyk holds a B.Eng(1997) degree and a M.Eng(1999) degree (with specialization in xDSL technology) from the University of Pretoria. He is currently busy with a PhD degree at the University of Pretoria, looking into methods to improve performance and mitigate multiuser interference in ADSL and VDSL systems. He is an employee of Telkom SA Ltd. since 1997.

Louis P. Linde holds a Hons-BEng(1973) degree in Electrotechnical Engineering from the University of Stellenbosch and M.Eng (1980) and D.Eng (1983) degrees in Electronic Engineering from the University of Pretoria. He is presently the Group Head of Signal Processing and Telecommunications in the Department of Electrical, Electronic and Computer Engineering (E,E&C Eng), Faculty of Engineering, University of Pretoria, as well as Director of both the Centre for Radio and Digital Communication (CRDC) and the RaDiCom Group in RE at UP.